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On the Application of Palm Probability for Obtaining the Waiting Time Distribution Between the First and Higher Order Conceptions

1. Introduction

SINGH (1964) has modelled a probability distribution on the time of first birth. Assuming that a conception taking place during the fecundable period following the marriage with hazard rate A, the waiting time distribution for the first conception is given by

$$f(t) = \lambda e^{-\lambda t} \qquad 0 \leqslant t < \infty \tag{O)}$$

On further Assumption, that the fecundability parameter A varies from individual to individual (even with the same parity group) following a probability density given by

$$\phi(\lambda) = \frac{a^k}{\Gamma(k)} e^{-a\lambda} \lambda^{k-1} \qquad a, k > 0 \tag{II}$$

the waiting time distribution for the first conception following the effective marriage is modelled by Singh (1964) as

$$\psi(t) = \frac{ak^a}{(k+t)^{a+1}}, \qquad a, k > 0 \tag{III}$$

However, the problem of obtaining the waiting time distribution between the first and the second conception or between the first and the nth conception ($n = 2, 3 \ldots\ldots\ldots$) can not be obtained as a direct generalization of the result obtained by Singh (1964); because on account of the weighting of the Poisson Process by a Gamma distribution the renewal structure of the process is completely destroyed leading to inter-arrival distributions having infinitely divisible structures with dependent increments. (Biswas and Pachal, 1983).

Denoting by $X(t)$ the number of compound Poisson events (weighted by a Gamma distribution given by (ii) it follows that for all $t > s$

$$\begin{aligned} Cov\,(X(S),\, X(t) - X(S)) &\\ &= E_\lambda[X(S)\,(X(t) - X(S) \mid \lambda)] - [E(\lambda)]^2\, S(t-S) \\ &= E_\lambda(\lambda^2 S(t-S)) - [E(\lambda)]^2\, S(t-S) \\ &= S(t-S)\,[[E(\lambda^2)] - [E(\lambda)]^2] = S(t-S)\; Var\,(\lambda) \qquad \textbf{(IV)} \end{aligned}$$

This makes the problem of obtaining the probability distribution of the inter-arrival in a compound Poisson process (or any such compound dependent process) more complicated. On the other hand, this is the type of distribution we require while obtaining the waiting time distribution between the first and the second conceptions. Using the concept of Palm Probability (Khintchine (1960)), Biswas and Pachal (1983) have however obtained the probability distribution of the 1st and the nth conception ($n = 2, 3, \ldots\ldots$) given that the first conception has occurred at time $T = 0$

$$f_n(t \mid \cdot) = \frac{t^{n-1}\, a^{k+1}\,(k+1)\,(k+2)\ldots(k+n-1)}{(n-1)!\,(a+t)^{k+n+1}} \qquad \textbf{(V)}$$

which is based on a similar assumption of having a time independent intensity of conception λ and assuming λ to conform to a distribution as in (ii) (proof vide appendix A).

For $n = 1$ i.e., the probability of the waiting time for second conception subject to the first conception taking place at $T = 0$ is, therefore, given by

$$f_1(t \mid \cdot) = \frac{a^{k+1}\,(k+1)}{(a+t)^{k+2}} \qquad \textbf{(VI)}$$

It may be noted that the probability distribution of the waiting time between the effective marriage and the first complete conception (given by iii) is obtainable from the corresponding distribution of the waiting time between the first and the second conception given by (vi) replacing the parameter K + I to K and vice versa.

The present paper is devoted to show a Demographic application of the Palm Probability technique by constructing the monthly distribution of the waiting time of 2nd conception given that the first conception for a cohort taking place at a time $T = 0$. Such procedures can easily be extended for obtaining the waiting time distribution between (n-l)th and nth conception using the same technique.

2. Methodology for Constructing the Monthly Probability Distribution for the Second Order of Conception for the Cohort (having the First Conception at : $T = 0$).

Assumptions

Following the first conception taking place at $T = 0$ (for the cohort), we assume that the entire cohort is exposed to the risk of further conception following an infecundable period which is composed of 9 months of Gestation period (for a full term live birth) and an average period of 3 months of Post Partum Amenerrhoea.

Hence $\pi = 1$ (in yearly unit).

Now

$$\tilde{f}_1 (t \mid \cdot) = (a + \pi)^{k+1} \frac{(k+1)}{(a+t)^{k+2}} ; \pi < t < \infty$$
$$= 0 \text{ Otherwise} \qquad \text{(VII)}$$

Where $\tilde{f}_1 (t \mid \cdot)$ is the distribution corresponding to $f_1 (t \mid \cdot)$ (vide vi) with the origin shifted at π.

It may be seen that $\tilde{f}_1 (t \mid \cdot)$ is a proper probability distribution.

With this set up the probability of a conception taking place between is $\pi + j$ and $\pi + j + 1$ for $j = 0, 1, 2, \ldots$

$$P_{\pi+j} = \left[\int_{\pi+j}^{\pi+j+1} \frac{dt}{(a+t)^{k+2}} \right] (a + \pi)^{k+1} (k + 1)$$
$$= (a + \pi)^{k+1} \left[\frac{1}{(a + \pi + j)^{k+j}} - \frac{1}{(a + \pi + j + 1)^{k+1}} \right]$$
$$\text{for } \pi < t < \infty \qquad \text{(VIII)}$$

Mean time for the first conceptive delay from marriage

$$E_1 (T) = ka^k \int_0^{\infty} \frac{t\, dt}{(a+t)^{k+1}} = \frac{a}{k-1} \qquad \text{(IX)}$$

The mean between the 1st and the 2nd conceptions is given by

$$E_2 (T) = \int_{\pi} t f_1 (t \mid \cdot)\, dt$$
$$= \frac{\pi (k + 1) + a}{k} \qquad \text{(X)}$$

3. Estimation of the Parameters

While estimating the parameters a and k, it has been found to be advisable to take into account of the ratio of the $E_1(T)$ and $E_2(T)$ rather than estimate a and k using the method of moments based on the data of the waiting time distribution between the first and the second conception only. This indeed does not use the information available from the distribution for the 1st conceptive delay from marriage. On the other hand it is of considerable demographic significance to employ the information provided by the waiting time distribution of the first conceptive delay while obtaining the waiting time distribution between the 1st and 2nd order of conceptions. This is based on the tacit assumption that the inter-arrival distribution between the first and the second order conceptions depend primarily on the waiting time between the effective marriage and the first order of conception. (They may be connected by a relation of markovity).

With this assumptions

$$\frac{E_1(T)}{E_2(T)} = \left[\frac{a/k - 1}{\pi(k+1) + a/k}\right] = \chi_0 \text{ say} \tag{XI}$$

Taking $\pi = 1 \Rightarrow$

$$a = \frac{\chi_0(k^2 - 1)}{k} \Big/ \left[1 - \frac{\chi_0(k-1)}{k}\right] \tag{XII}$$

Also using Srinivasan's (1972) data quoted by Mitra and Banerjee (1982), which gives the average interval between the first and the second birth for a class of women in South India (which is the same as the mean waiting time between the first and the second complete conceptions) we have

$$\frac{(k+1)+a}{k} \simeq 3 \qquad (\because \pi = 1)$$

$$\Rightarrow K = \frac{a+1}{2} \tag{XIII}$$

(xii) — (xiii) $\Rightarrow$

$$\chi_0 k^2 - \chi_0 - ak + a\chi_0(k-1) = 0$$

$$\Rightarrow k^2(\chi_0) - (2k-1)\,k\,(1-\chi_0) - 2k\chi_0 = 0 \tag{XIV}$$

Putting $a = 2k - 1$ from (xiii) in (xiv) a Quadratic equation in terms of K only as

$$k^2\chi_0 - [2k^2(1-\chi_0) - k(1-\chi_0)] - 2k\chi_0 = 0$$

$$\Rightarrow k = \left(\frac{3\chi_0 - 1}{3\chi_0 - 2}\right) \tag{XV}$$

An estimate of Xo was obtained using the data available from Table 58 of Fertility Differentials in India (1982) Vital Statistics Division of the office of the Registrar General of India. The procedure is as follows :

The mean interval between the effective marriage and the first conception was obtained from that of the mean interval between the effective *marriage* and the first birth by subtracting from the latter *b* the gestation period (= 9 months) following a conception; whereas the mean interval between the first *b* and the second conception was taken to be the same as that of the mean interval between the first and the second order of live births. As the above figures were provided with separate rural and urban breakdown, therefore combined pooled estimates for both the intervals were computed using the corresponding ever married women in the fecundable age groups as weights.

An estimate of x_0 came out to be

$$\hat{x}_0 = \frac{25.74}{34.34} = 0.7496$$

which provided the corresponding estimate of *K* (using (XV)) as $\hat{K}$ = 5.021086

Using (XIII) we have a = 9.0422.

4. Results

Using the above estimate of a and *k* the probability of having a reconception following the expiry of infccundable period after the 1st conception has been computed using the result (viii). The results are summarised in Table 1.

TABLE 1- SHOWING THE MONTHLY PROBABILITY OF CONCEPTION **FROM** HYPOTHETICAL COHORT HAVING A FIRST CONCEPTION AT A FIXED TIME (T = 0).

(X_0 = 0.7496; $\hat{k}$ = 5.0211; $\hat{a}$ = 9.0421;)

Month after the expiry of the infecundable period (1 year) following the first conception	*Monthly Probability of Conception*	*Cumulative Probability upto the given month*
I	2	3
0	0.0485	0.0485
1	0.0458	0.0943
2	0.0432	0.1375
3	0.0409	0.1784

Table *I (contd. on page* 282)

Table 1 *(contd. from* ***page*** 281)

I	*2*	*3*
4	0.0386	0.2170
5	0.0365	0.2535
6	0.0345	0.2880
7	0.0327	0.3207
5	0,0309	0.3516
9	0.0293	0.3809
10	0.0278	0.4087
11	0.0263	0.4350
12	0.0249	0.4599
13	0.0237	0.4836
14	0.0225	0.5061
15	0.0213	0.5274
16	0.0203	0.5477
17	0.0193	0.5670
18	O.OH3	0.5853
19	0.0174	0.6027
20	0.0166	0.6193
21	0.0158	0.6351
22	0.0150	0.6501
23	0.0143	0.6644
24	0.0136	0.6780
25	0.0129	0.6909
26	0.0124	0.7033
27	0.0118	0.7151
28	0.0113	0.7264
29	0.0107	0.7371
30	0.0102	0.7473
31	0.0098	0.7571

Table *1 (contd. on page* 283)

Table 1 (*coatd- from page*'232)

1	2	3
32	0.0093	0.7664
33	0.0089	0.7753
34	0.0085	0.7838
35	0.0081	0.7919
36	0.0078	0.7997
37	0.0074	0.8071
38	0.0071	0.8142
39	0.0068	0.8210
40	0.0065	0.8275
41	0.0063	0.8338
42	0.0060	0.8398
43	0.0057	0.8455
44	0.0055	0.8510
45	0.0053	0.8563
46	0.0051	0.8614
47	0.0048	0.8662
48	0.0046	0.8708
49	0,0045	0.8753
50	0.0043	0.8796
51	0.0041	0.8837
52	0.0039	0.8876
53	0.0038	0.8914
54	0.0036	0.8950
55	0.0035	0.8985
56	0.0034	0.9019
57	0.0032	0.9051
58	0.0031	0.9082

Table 1 (*contd. on page* 284)

Table 1 *(contd. from Page* 283)

1	2	3
59	0.0030	0.9112
60	0.0029	0.9141
61	0.0030	0.9171
62	0.0026	0.9197
63	0.0025	0.9222
64	0.0025	0.9247
65	0.0024	0.9271
66	0.0023	0.9294
67	0.0022	0.9316
68	00021	0-9337
69	0.0020	0.9357
70	0.0020	0.9377
71	O.0019	0.9396
72	0.0018	0.9414
73	0.0018	0.9432
74	0.0017	0.9449
7.5	0.C016	0-9465
76	0.0016	0.9431
77	0.0015	0.9496
78	0.0015	0.9511
79	0.0014	0.9525
80	0.0014	0.9539
81	O.0013	0.9552
82	0.C013	0.9565
83	0.0012	0.9577
84	0.0012	0.9589
85	0.0012	0.9601

Table 1 *(cantd. on page* 285)

Table 1 *(contd. from page* 284)

1	*2*	*3*
86	0.0011	0.9612
87	0.0018	0.9630
88	0.0010	0.9640
89	0.0010	0.9650
90	0.0009	0.9659
91	0.0009	0.9669
92	0.0009	0.9677
93	0.0008	0.9685
94	0.0008	0.9693
95	0.0008	0.9701
96	0.0008	0.9709
97	0.0008	0.9717
98	0.0007	0.9724
99	0.0007	0.9731
100	0.0007	0.9738
101	0.0006	0.9744
102	0.0006	0.9750
103	0.0006	0.9756
104	0.0006	0.9762
705	0.0006	*0.9763*
106	0.0005	0.9773
107	0.0005	0.9778
108	0.0005	0.9783
109	0.0005	0.9788
110	0.0005	0.9793
111	0.0005	0.9798
112	00005	0.9803

Table 1 *(contd. on page* 286}

Table 1 *(contd. from page* 285)

1	2	3
113	0.0005	0.9808
114	0.0004	0.9812
115	0.0004	0.9816
116	0.0004	0.9820
117	0.0004	0.9824
118	0.0004	0.9828
119	0.0004	0.9832

Similar tables have also been constructed using hypothetical values of x_0 and the corresponding estimated values of *a* and *k* as follows

TABLE 2—ESTIMATED VALUES OF *a* AND *k* CORRESPONDING TO DIFFERENT HYPOTHETICAL VALUES OF X_0,

X_0	0.7	0.8	0.9	1.0
$\hat{a}$	21.0	6.0	3.8571	3.0
$\hat{k}$	*11.0*	3.5	2.4286	2.0

These tables arc given in Appendix B.

5. Discussion

An overall glance in all the tables shows that the conception rate monotonically decreases and by a period of 120 months following the date of first conception about 98% of the women get reconceived. The Quartiles of the period of reconception following the first conception are as follows :

First Quartile = 17 months
Second Quartile = 26 months
Third Quartiles = 43 months

It may further be noted that the rate of conception decreases very sharply in the initial stage and then the rate of decreases slows down and after 120 months or so the decrease is very small and asymptotic.

As mentioned earlier similar procedures may be adopted to construct tables for the waiting times between 2-3, 3-4, . . . order of conceptions. Such tables are Very useful in studying the fertility behaviour and therefore useful in the demographic planning.

References

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APPENDIX A

Derivation of the density function

$$f_n(t \mid \cdot) = \frac{t^{n-1} a^{k+1} (k+1)(k+2) \ldots (k+n-1)}{(n-1)!\,(a+t)^{k+n+1}}$$

We define $\phi_r(t)$ as the conditional probability of r events ($r = 0, 1, 2, \ldots$) in $(0, t]$ given that an event (say the first event) has occurred at time $T = 0$. Khintchine (1960) gives

$$V_r(t) = \frac{k}{a} \int_0^t [\phi_{r-1}(t) - \phi_r(t)]\, d\tau \qquad \text{(A1)}$$

and

$$V_0(t) = 1 - \frac{k}{a} \int_0^t \phi_0(\tau)\, d\tau \qquad \text{(A2)}$$

where $V_r(t)$ represents the unconditional probability of r number of events (arrivals) in $(0, t)$ following a Weighted Poisson Process (weighted by Gamma distribution as in (ii) in the text) and

$$\frac{k}{a} = \lim_{t \to 0} \frac{1 - V_0(t)}{t} \qquad \text{(A3)}$$

where k/a is the intensity of the compound Poisson process.

Then

$$V_0(t) = E_\lambda(e^{-\lambda t} \mid \lambda) = \int_0^\infty e^{-\lambda t} \psi(\lambda)\, d\lambda$$

$$= \frac{a^k}{\Gamma(k)} \int_0^\infty e^{-\lambda t}\, e^{-a\lambda}\, \lambda^{k-1}\, d\lambda$$

$$= \frac{a^k}{(a+t)^k} \qquad \text{(A4)}$$

Using (A2) and (A4) we have

$$\left(\frac{a}{a+t}\right)^k = 1 - \frac{k}{a} \int_0^t \phi_0(\tau)\, d\tau \qquad \text{(A5)}$$

Which on differentiation gives

$$\frac{a^{k+1}}{(a+t)^{k+1}} = \phi_0(t) \qquad \text{(A6)}$$

Again $\phi_0(t) = P[T_1 > t \mid \text{an event has occurred at } T = 0]$
Where T_1 is the random time of the first event following the occurrence of an event at $T = 0$.
Hence

$$f_1(t \mid \cdot) = \frac{d}{dt}\,(1 - \phi_0(t)) = \frac{(k+1)\,a^{k+1}}{(a - t)^{k+2}} \tag{A7}$$

Here $f_1(t \mid \cdot)$ is the conditional density of the waiting time of the second event given that the event has occurred at $T = 0$.
(A7) is obviously different from the unconditional density of the first arrival (given in (iii) of the text).

Proceeding in this way, putting $r = 1$ and differentiating both sides of (A1)

$$V_1(t) = \frac{k}{a}\,[\phi_0(t) - \phi_1(t)]$$

$$\Rightarrow \frac{d}{dt}\left[k\left(\frac{a}{a+t}\right)^{k}\left(\frac{t}{a+t}\right)\right] = \frac{k}{a}\left[\frac{a^{k+1}}{(a+t)^{k+1}} - \phi_1(t)\right]$$

$$\Rightarrow (\phi_1(t))' = \frac{(k+1)\,a^{k+1}\,t}{(a+t)^{k+2}} \tag{A8}$$

Then

$$F_2(t \mid \cdot) = 1 - \phi_0(t) - \phi_1(t) \tag{A9}$$

Where $F_2(t \mid \cdot)$ is the conditional cumulative distribution (c.d.f.) of the waiting time of the second arrival given that at $T = 0$ the first arrival took place.
Then

$$f_2(t \mid \cdot) = \frac{a^{k+1}\,(k+1)}{(a+t)^{k+2}} - \frac{a^{k+2}\,(k+2)}{(a+t)^{k+3}}$$

$$- \frac{ka^{k+1}}{(a+t)^{k+2}} + \frac{k(k+2)\,ta^{k+1}}{(a+t)^{k+3}} + \frac{a^{k+1}\,(k+1)}{(a+t)^{k+2}}$$

$$= \frac{ta^{k+1}\,(k+1)\,(k+2)}{(a+t)^{k+3}} \tag{A10}$$

Which is the inter arrival density function between the first and the third arrival given that the first arrival took place at $T = 0$ in compound Poisson process.

Proceeding precisely in the same way

$$V_2'(t) = \frac{k}{a}\,[\phi_1(t) - \phi_2(t)] \tag{A11}$$

$$\frac{d}{dt}\left\{\frac{(k+1)k}{2}\left(\frac{a}{a+t}\right)^k\left(\frac{t}{a+t}\right)^2\right\}$$

$$= \frac{k}{a}\left[\frac{(k+1)\,a^{k+1}}{(a+t)^{k+1}} - \phi_2(t)\right]$$

$$\Rightarrow \phi_2(t) = \frac{t^2 a^{k+1}\,(k+1)\,(k+2)}{2\,!\,(a+t)^{k+1}} \tag{A12}$$

$$\Rightarrow F_3(t) = 1 - [\phi_0(t) + \phi_1(t) + \phi_2(t)]$$

$$= 1 - \left[\frac{a^{k+1}\,(k+1)\,(k+2)}{2\,!\,(a+t)^{k+3}} + \frac{a^{k+1}\,t\,(k+1)}{(a+t)^{k+2}} + \frac{a^{k+1}}{(a+t)^{k+1}}\right]$$

Where we get

$$f_3(r \mid \cdot) = \frac{a^{k+1}\,(k+2)\,(k+3)}{2\,!\,(a+t)^{k+4}}$$

Using

$$f_n(t \mid \cdot) = \frac{d}{dt}\left\{1 - \sum_{r=0}^{n} \phi_r(t)\right\}$$

and the recurrence relation

$$f_n(t \mid \cdot) = f_{n-1}(t \mid \cdot) - \phi'_{n-2}(t) + \frac{a}{k}\,V''_{n-1}(t) \tag{A13}$$

We have

$$f_n(t \mid \cdot) = \frac{t^{n-1}\,a^{k+1}\,(k+1)\,(k+2)\,\ldots\,(k+n)}{(n-1)\,!\,(a+t)^{k+n+1}} \tag{A14}$$

Which provides the waiting time distribution between the first and the $(n+1)$th arrival given that the first arrival has occurred at $T = 0$ for a compound Poisson process weighted by a Gamma distribution.

APPENDIX B

TABLE B 1—SHOWING THE MONTHLY PROBABILITY OF CONCEPTION FROM HYPOTHETICAL COHORT HAVING A FIRST CONCEPTION AT A FIXED TIME ($T = 0$)
($\chi_0 = 0.7$ $\hat{a} = 21$ $\hat{k} = 11$)

Month after the expiry of the infecundable period (1 year) following the first conception	*Monthly Probability of Conception*	*Cumulative Probability upto the given month*
(*1*)	(*2*)	(*3*)
0	0.0443	0.0443
1	0.0422	0.0865
2	0.0402	0.1267
3	0.0383	0.1650
4	0.0365	0.2015
5	0.0347	0.2362
6	0.0331	0.2693
7	0.0316	0.3009
8	0.0301	0.3310
9	0.0287	0.3597
10	0.0274	0.3871
11	0.0261	0.4132
12	0.0249	0.4381
13	0.0238	0.4619
14	0.0227	0.4846
15	0.0216	0.5062
16	0.0207	0.5269
17	0.0197	0.5466
18	0.0188	0.5654
19	0.0180	0.5834
20	0.0172	0.6006

Table B 1 (*contd. on page* 292)

Table B 1 (*contd. from page* 291)

1	2	3
21	0.0164	0.6170
22	0.0157	0.6327
23	0.0150	0.6477
24	0.0143	0.6620
25	0.0137	0.6757
26	0.0131	0.6888
27	0.0125	0.7013
28	0.0120	0.7133
29	0.0115	0.7248
30	0.0110	0.7358
31	0.0105	0.7463
32	0.0100	0.7563
33	0.0096	0.7659
34	0.0092	0.7751
35	0.0088	0.7839
86	0.0084	0.7923
37	0.0081	0.8004
38	0.0077	0.8081
39	0.0074	0.8155
40	0.0071	0.8226
41	0.0068	0.8294
42	0.0065	0.8359
43	0.0062	0.8421
44	0.0060	0.8481
45	0.0057	0.8538
46	0.0055	0.8593
47	0.0053	0.8646
48	0.0051	0.8697

Table B 1 (*contd. on page* 293)

Table B 1 (*contd. from page* 292)

1	*2*	*3*
49	0.0049	0.8746
50	0.0047	0.8793
51	0.0045	0.8838
52	0.0043	0.8881
53	0.0041	0.8922
54	0.0039	0.8961
55	0.0038	0.8999
56	0.0036	0.9035
57	0.0035	0.9070
58	0.0034	0.9104
59	0.0032	0.9136
	0.0031	0.9167
61	0.0029	0.9196
62	0.0028	0.9224
63	0.0027	0.9257
64	0.0026	0.9297
65	0.0025	0.9302
66	0.0024	0.9326
67	0.0023	0.9349
68	0.0022	0.9371
69	0.0021	0.9392
70	0.0020	0.9412
71	0.0028	0.9440
72	0.0019	0.9459
73	0.0018	0.9477
74	0.0017	0.9494
75	0.0017	0.9511
76	0.0016	0.9527

Table B 1 (*contd. on page* 294)

Table B 1 (*contd. from page* 293)

1	*2*	*3*
77	0.0016	0.9543
78	0.0015	0.9558
79	0.0014	0.9572
80	0.0014	0.9586
81	0.0013	0.9599
82	0.0013	0.9612
83	0.0012	0.9624
84	0.0012	0.9636
85	0.0012	0.9648
86	0.0011	0.9659
87	0.0011	0.9670
88	0.0010	0.9680
89	0.0010	0.9690
90	0.0009	0.9699
91	0.0009	0.9708
92	0.0009	0.9717
93	0.0008	0.9725
94	0.0008	0.9733
95	0.0008	0.9741
96	0.0007	0.9748
97	0.0007	0.9755
98	0.0007	0.9762
99	0.0006	0.9768
100	0.0006	0.9774
101	0.0006	0.9780
102	0.0006	0.9786
103	0.0006	0.9792

Table B 1 (*contd. on page* 295)

Table B.1 (*contd. from page* 294)

1	*2*	*3*
104	0.0006	0.9748
105	0.0006	0.9804
106	0.0005	0.9809
107	0.0005	0.9814
108	0.0005	0.9819
109	0.0005	0.9824
110	0.0004	0.9828
111	0.0004	0.9832
112	0.0004	0.9836
113	0.0004	0.9840
114	0.0004	0.9844
115	0.0004	0.9848
116	0.0003	0.9851
117	0.0003	0.9854
118	0.0003	0.9857
119	0.0003	0.9860

TABLE B 2 : SHOWING THE MONTHLY PROBABILITY OF CONCEPTION FROM HYPOTHETICAL COHORT HAVING A FIRST CONCEPTION AT A FIXED TIME (T = 0)
($\chi_0 = 0.8\ \hat{a} = 6\ \hat{k} = 3.5$)

Month after the expiry of the infecundable period (1 year) following the first conception	*Monthly Probability of Conception*	*Cumulative Probability upto the given month*
1	*2*	*3*
0	0.0518	0.0518
1	0.0486	0.1004
2	0.0456	0.1460
3	0.0428	0.1888
4	0.0402	0.2290
5	0.0378	0.2668
6	0.0355	0.3023
7	0.0335	0.3358
8	0.0315	0.3673
9	0.0297	0.3970
10	0 0280	0.4250
11	0.0264	0.4514
12	0.0249	0.4763
13	0.0236	0.4999
14	0.0223	0.5222
14	0.0211	0.5433
16	0.0199	0.5632
17	0.0189	0.5821
18	0.0179	0.6000
19	0.0169	0.6169
20	0.0161	0.6330
21	53	0.6483
22	0.0145	0·6728
23	0.0139	0.6767

Table B 2 (*contd. on page* 287)

Table B 2 (*contd. from page* 296)

1	*2*	*3*
24	0 0131	0.6898
25	0.0125	0.7023
26	0.0119	0.7142
27	0.0113	0.7255
28	0.0107	0.7362
29	0.0102	0.7464
30	0.0097	0.7561
31	0.0093	0.7654
32	0.0089	0.7743
33	0.0085	0.7828
34	0.0081	0.7909
35	0.0077	0.7986
36	0.0073	0.8059
37	0.0070	0.8129
38	0.0067	0.8196
39	0.0064	0.8260
40	0.0061	0.8321
41	0.0058	0.8379
42	0.0056	0.8435
43	0.0054	0.8489
44	0.0052	0.8541
45	0.0049	0.8590
46	0.0047	0.8637
47	0.0046	0.8683
48	0.0044	0.8727
49	0.0042	0.8769
50	0.0040	0.8809

Table B 2 (*contd. on page* 298)

Table B 2 (*contd. from page* 297)

1	*2*	*3*
51	0.0038	0.8847
52	0.0037	0.8884
53	0.0036	0.8920
54	0.0034	0.8954
55	0.0033	0.8987
56	0.0032	0.9019
57	0.0030	0.9049
58	0.0029	0.9078
59	0.0028	0.9106
60	0.0027	0.9133
61	0.0026	0.9159
62	0.0025	0.9184
63	0.0024	0.9208
64	0.0023	0.9231
65	0.0022	0.9253
66	0.0022	0.9275
67	0.0021	0.9296
68	0.0020	0.9316
69	0.0019	0.9335
70	0.0019	0.9354
71	0.0018	0.9372
72	0.0017	0.9389
73	0.0017	0.9406
74	0.0016	0.9422
75	0.0016	0.9438
76	0.0015	0.9453
77	0.0014	0.9467

Table B 2 (*contd. on page* 299)

Table B 2 (*contd. from page* 298)

1	*2*	*3*
78	0.0014	0.9481
79	0.0013	0.9494
80	0.0013	0.9507
81	0.0012	0.9519
82	0.0012	0.9531
83	0.0012	0.9543
84	0.0011	0.9554
85	0 0011	0.9565
86	0.0010	0.9575
87	0.0010	0.9585
88	0.0010	0.9595
89	0.0009	0.9604
90	0.0009	0.9613
91	0.0009	0.9622
92	0.0009	0.9631
93	0.0008	0.9639
94	0.0008	0.9647
95	0.0008	0.9655
96	0.0007	0.9662
97	0.0007	0.9669
98	0.0007	0.9676
99	0.0007	0.9683
100	0.0007	0.9690
101	0.0006	0.9696
102	0.0006	0.9702
103	0.0006	0.9708
104	0.0006	0.9714

Table B 2 (*contd. on page* 300)

Table B 2 (*contd. from page 299*)

1	2	*3*
105	0.0006	0.9720
106	0.0005	0.9725
107	0.0005	0.9730
108	0.0005	0.9735
109	0.0005	0.9740
110	0.0005	0.9745
111	0.0005	0.9750
112	0.0004	0.9754
113	0.0004	0.9758
114	0.0004	0.9762
115	0.0004	0.9766
116	0.0004	0.9770
117	0.0004	0.9774
118	0.0004	0.9778
119	0.0004	0.9782

TABLE B 3—SHOWING THE MONTHLY PROBABILITY OF CONCEPTION FROM HYPOTHETICAL COHORT HAVING A FIRST CONCEPTION AT A FIXED TIME ($T=0$)

(χ_0 Rs = 9 $\hat{a}$ = 3.857142 $\hat{k}$ = 2.428571)

Month after the expiry of the infecundable period (1 year) following the first conception	*Monthly probability of Conception*	*Cumulative Probability upto the given month*
1	*2*	*3*
0	0.0566	0.0566
1	0.0525	0.1091
2	0.0488	0.1579
3	0.0454	0.2033
4	0.0423	0.2456
5	0.0394	0.2850
6	0.0368	0.3218
7	0.0344	0.3562
8	0.0322	0.3884
9	0.0301	0.4185
10	0.0282	0.4467
11	0.0264	0.4731
12	0.0248	0.4979
13	0.0233	0.5212
14	0.0219	0.5431
15	0.0206	0.5637
16	0.0194	0.5831
17	0.0183	0.6014
18	0.0173	0.6187
19	0.0163	0.6350
20	0.0154	0.6504
21	0.0146	0.6650
22	0.0138	0.6788

Table B 3 (*contd. on page* 302)

Table B 3 (*contd. from page* 301)

1	*2*	*3*
23	0.0131	0.6919
24	0.0124	0.7043
25	0.0117	0.7160
26	0.0111	0.7271
27	0.0106	0.7377
28	0.0100	0.7477
29	0.0095	0.7572
30	0.0091	0.7663
31	0.0086	0.7749
32	0.0082	0.7831
33	0.0078	0.7909
34	0.0075	0.7984
35	0.0071	0.8055
36	0.0068	0.8123
37	0.0065	0.8188
38	0.0062	0.8250
39	0.0059	0.8309
40	0.0056	0.8365
41	0.0054	0.8419
42	0.0052	0.8471
43	0.0049	0.8520
44	0.0048	0.8568
45	0.0046	0.8614
46	0.0044	0.8658
47	0.0042	0.8700
48	0.0040	0.8740
49	0.0039	0.8779
50	0.0037	0.8816

Table B 3 (*contd. on page* 303)

Table B 3 (*contd. from page* 302)

1	*2*	*3*
51	0.0036	0.8852
52	0.0034	0.8886
53	0.0033	0.8919
54	0.0032	0.8951
55	0.0030	0.8981
56	0.0029	0.9010
57	0.0028	0.9038
58	0.0027	0.9065
59	0.0026	0.9091
60	0.0025	0.9116
61	0.0024	0·9140
62	0.0023	0.9163
63	0.0022	0.9185
64	0.0022	0.9207
65	0.0021	0.9228
66	0.0020	0.9248
67	0.0019	0.9267
68	0.0019	0.9286
69	0.0018	0.9304
70	0.0018	0.9322
71	0.0017	0.9339
72	0.0016	0.9355
73	0.0016	0.9371
74	0.0015	0.9386
75	0.0015	0.9401
76	0.0014	0.9415
77	0.0014	0.9429

Table B 3 (*contd. on page* 304)

Table B 3 (*contd. from page* 309)

1	*2*	*3*
78	0.0013	0.9442
79	0.0013	0.9455
80	0.0012	0.9467
81	0.0012	0.9479
82	0.0011	0.9490
83	0.0011	0.9501
84	0.0011	0.9512
85	0.0010	0.9522
86	0.0010	0.9532
87	0.0010	0.9542
88	0.0009	0.9551
89	0.0009	0.9560
90	0.0009	0.9569
91	0.0008	0·9577
92	0.0008	0.9585
93	0.0008	0.9593
94	0.0008	0.9601
95	0.0008	0.9609
96	0.0007	0.9616
97	0.0007	0.9623
98	0.0007	0.9630
99	0.0007	0.9637
100	0.0006	0.9643
101	0.0006	0.9649
102	0.0006	0.9655
103	0.0006	0.9661
104	0.0006	0.9667

Table B 3 (*contd. on page* 305)

Table B 3 (*contd. from page* 304)

1	*2*	*3*
105	0.0006	0·9673
106	0.0005	0.9678
107	0.0005	0.9683
108	0.0005	0.9688
109	0.0005	0.9693
110	0.0005	0.9698
111	0.0005	0.9703
112	0.0005	0.9708
113	0.0004	0.9712
114	0.0004	0.9716
115	0.0004	0.9720
116	0.0004	0.9724
117	0.0004	0.9728
118	0.0004	0.9732
119	0.0004	0.9736

TABLE B 4—SHOWING THE MONTHLY PROBABILITY OF CONCEPTION FROM HYPOTHETICAL COHORT HAVING A FIRST CONCEPTION AT A FIXED TIME ($T = 0$)

($\chi_0 = 1$ $a = 3$ $\hat{k} = 2$)

Month after the expiry of the infecundable period (1 year) following the first conception	*Monthly Probability of Conception*	*Cumulative Probability upto the given month*
1	*2*	*3*
0	0.0599	0.0599
1	0.0553	0.1152
2	0.0510	0.1662
3	0.0471	0.2133
4	0.0436	0.2569
5	0.0405	0.2974
6	0.0376	0.3350
7	0.0349	0.3699
8	0.0326	0.4025
9	0.0303	0.4328
10	0.0283	0.4611
11	0.0264	0.4875
12	0.0248	0.5123
13	0.0232	0.5355
14	0.0217	0.5572
15	0.0204	0.5776
16	0.0192	0.5968
17	0.0180	0.6148
18	0.0169	0.6317
19	0.0159	0.6476
20	0.0151	0.6627
21.	0.0142	0.6769

Table B 4 (*contd. on page* 307)

Table B 4 (*contd. from page* 306)

1	*2*	*3*
22	0.0134	0.6903
23	0.0126	0.7029
24	0.0120	0.7149
25	0.0114	0.7263
26	0.0107	0.7370
27	0.0102	0.7472
28	0.0096	0.7568
29	0.0092	0.7660
30	0.0087	0.7747
31	0.0083	0.7830
32	0.0078	0.7908
33	0.0075	0.7983
34	0.0072	0.8055
35	0.0068	0.8123
36	0.0065	0.8188
37	0.0062	0.8250
38	0.0059	0.8309
39	0.0056	0.8365
40	0.0054	0.8419
41	0.0052	0.8471
42	0.0049	0.8520
43	0.0047	0.8567
44	0.0045	0.8612
45	0.0043	0.8655
46	0.0041	0.8696
47	0.0039	0.8735
48	0.0038	0.8773

Table B 4 (*contd. on page* 308)

Table B 4 (*contd. from page* 307)

1	*2*	*3*
49	0.0036	0.8809
50	0.0035	0.8844
51	0.0035	0.8879
52	0.0032	0.8911
53	0.0031	0.8942
54	0.0030	0.8972
55	0.0028	0.9000
56	0.0027	0.9027
57	0.0026	0.9053
58	0.0025	0.9078
59	0.0024	0.9102
60	0.0023	0.9125
61	0.0023	0.9148
62	0.0022	0.9170
63	0.0021	0.9191
64	0.0021	0.9212
65	0.0019	0.9231
66	0.0019	0.9250
67	0.0018	0.9268
68	0.0018	0.9286
69	0.0017	0.9303
70	0.0016	0.9319
71	0.0016	0.9335
72	0.0016	0.9351
73	0.0015	0.9368
74	0.0014	0.9380
75	0.0014	0.9394

Table B 4 (*contd. on page* 309)

Table B 4 (*contd. from page* 309)

1	*2*	*3*
76	0.0013	0.9407
77	0.0013	0.9420
78	0.0012	0.9432
79	0.0012	0.9444
80	0.0012	0.9456
81	0.0011	0.9467
82	0.0011	0.9478
83	0.0011	0.9489
84	0.0010	0.9499
85	0.0010	0.9509
86	0.0010	0.9519
87	0.0009	0.9528
88	0.0009	0.9537
89	0.0009	0.9546
90	0.0009	0.9555
91	0.0008	0.9563
92	0.0008	0.9571
93	0.0008	0.9579
94	0.0008	0.9587
95	0.0007	0.9594
96	0.0007	0.9601
97	0.0007	0.9608
98	0.0007	0.9615
99	0.0007	0.9622
100	0.0006	0.9628
101	0.0006	0.9634
102	0.0006	0.9640

Table B 4 (*contd. on page* 310)

Table B 4 (*contd. from page* 309)

1	*2*	*3*
103	0.0006	0.9646
104	0.0006	0.9652
105	0.0006	0.9658
106	0.0006	0.9664
107	0.0006	0.9670
108	0.0005	0.9675
109	0.0005	0.9680
110	0.0005	0.9685
111	0.0005	0.9690
112	0.0005	0.9695
113	0.0005	0.9700
114	0.0005	0.9705
115	0.0005	0.9710
116	0.0004	0.9714
117	0.0004	0.9718
118	0.0004	0.9722
119	0.0004	0.9726

APPENDIX C

Alternate derivation of the model

Let $V_k(t)$ = Probability of K events (number of conceptions in (0, t] given that at $T = 0$ an event (conception) has taken place (Palm Probability)

and

$\phi_k(t)$ = unconditional probability of K events in $(0, t)$.

The probability generating functions (p.g.f.) $G(g, t)$ and $G_0(z, t)$ of $V_k(t)$ and $\phi_k(t)$ respectively are as follows :

$$G(z, t) = \sum_{k=0}^{\infty} V_k(t)\, z^k \tag{C 1}$$

$$G_0(z, t) = \sum_{k=0}^{\infty} \phi_k(t)\, z^k \tag{C 2}$$

Then

$$G(z, t) = \left(\frac{a}{a + t\,(1 - z)}\right)^k \tag{C 3}$$

$$G_0(z, t) = \left(\frac{a}{a + t\,(1 - z)}\right)^{k+1} \tag{C 4}$$

We have

$$\frac{\partial}{\partial t}\, G(z, t) = -\frac{k}{a}\,(1 - a)\, G_0(z, t) \tag{C 5}$$

(Vide Cox and Isham (1930)—Point Processes, Chapman and Hall, London, page 30)

If T_n is the time to nth event given that the first one has happened at $T = 0$ then

$$F_n(t) = P(T_n \leqslant t) = \sum_{k=n}^{\infty} \phi_k(t) \tag{C 6}$$

$$H_0(z, t) = \sum_{n=0}^{\infty} Z^n\, F_n(t) = \frac{1 - z\, G_0(z, t)}{1 - z} \tag{C 7}$$

and if

$f_n(t) = dF_n(t)/dt$ be the density function corresponding to $(n + 1)$th arrival given that the first arrival is at $T = 0$

then

$$\sum_{n=0}^{\infty} Z^n f_n(t) = \frac{-z\, \partial/\partial t\, G_0(z, t)}{1 - z} \tag{C 8}$$

$$= \frac{z(k+1)}{a} \left(\frac{a}{a + t(1-z)}\right)^{k+1}$$

$$= \sum_{J=1}^{\infty} \frac{z^J a^{k+1} \Gamma(J + K + 1)}{\Gamma(K+1)(a+1)^{k+J+1}(J-1)!} \tag{C 9}$$

Equating both sides, the coefficients of z we get $f_i(t)$ viz. the waiting time distribution of the second event given that the first has occured at $T = O$. Hence

$$f_1(t) = \frac{a^{k+1}\, \Gamma(k+2)}{\Gamma(k+1)(a+t)^{k+2}}$$

$$= \frac{a^{k+1}(k+1)}{(a+t)^{k+2}} \tag{C 10}$$

Changing the parameters a, k to a_i, k_i for the $(i + 1)$ *th* order of conception given that the ith conception has taken place at $T = O$ we have

$$f_i(t) = \frac{a_i^{k_i+1}(k_i+1)}{(a_i+t)^{k_i+2}} \quad 0, \leqslant t < \infty \tag{C 11}$$

and finally

$$f_n(t) = \frac{t^{n-1} a^{k+1}(k+1)\ldots(k+n)}{(n-1)!\,(a+t)^{k+n+1}} \tag{C 12}$$